CANTRELL PRIMARY AND NURSERY SCHOOL

WRITTEN CALCULATION POLICY

March 2022

Introduction

Children are introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas they develop ways of recording to support their thinking and calculation methods, use particular methods that apply to special cases, and learn to interpret and use the signs and symbols involved. Over time children learn how to use models and images, such as empty number lines, to support their mental and informal written methods of calculation. **As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to efficient written methods that can be used more generally**. By the end of Year 6 children are equipped with mental, written and calculator methods that they understand and can use correctly. When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy. At whatever stage in their learning, and whatever method is being used, it must still be underpinned by a secure and appropriate knowledge of number facts, along with those mental skills that are needed to carry out the process and judge if it was successful.

The aim is that by the end of Key Stage 2, the great majority of children will be able to use an efficient written method for each operation with confidence and understanding.

The written methods are sequenced showing progression through year groups, however if the children in your class are not ready for the next stage then introduce it when they are. Each new method should be introduced through guided group work and not whole class.

It is important that children's mental methods of calculation are practiced and secured alongside their learning and use of an efficient written method for addition, subtraction, multiplication and division.

Things to remember

- Children should always estimate first
- Always check the answer, preferably using a different method e.g. the inverse operation
- Always decide first whether a mental method is appropriate
- Pay attention to mathematical language
- Children who make persistent mistakes should return to the method that they can use accurately until ready to move on
- Children need to know number and multiplication facts by heart
- When revising or extending to harder numbers, refer back to expanded methods. This helps reinforce understanding and reminds children that they have an alternative to fall back on if they are having difficulties.
- Use practical equipment when introducing or reviewing a concept

Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways

End of FS2 through Year 1 and continuing in to Year 2

Stage 1: The empty number line

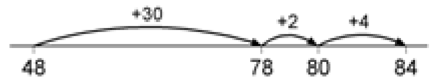
- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

Stage 1

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

8 + 7 = 15

48 + 36 = 84



or:

Year 2 progressing into Year 3

Stage 2: Partitioning

- The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.
- Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

Stage 2

Record steps in addition using partitioning:

47 + 76 = 47 + 70 + 6 = 117 + 6 = 123

47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123

Partitioned numbers are then written under one another:

$$\begin{array}{rcl} 47 & = & 40 + 7 \\ +76 & & 70 + 6 \\ \hline & & 110 + 13 = 123 \end{array}$$

Year 4 - the children need to be secure and confident in Point 2 of Stage 2, where the unit number is greater than 10 before introducing the column method. This method needs to be taught as guided group work with the children who are ready.

Year 4 onwards

Stage 4: Column method

- In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
- Later, extend to adding three two-digit numbers, two three digit numbers and numbers with different numbers of digits.

Stage 4

$$\begin{array}{r} 47 \\ +\ 76 \\ \hline 123 \\ \underline{11} \end{array} \qquad \begin{array}{r} 258 \\ +\ 87 \\ \hline 345 \\ \underline{11} \end{array} \qquad \begin{array}{r} 366 \\ +458 \\ \hline 824 \\ \underline{11} \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable

Written methods for subtraction of whole numbers

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact, f16 - 7, and their knowledge of place value;
- Partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

End of FS2, Year 1 and Year 2

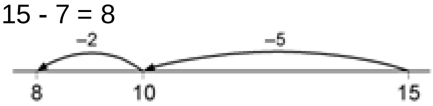
Stage 1: Using the empty number line

- The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten.
- The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47.
- With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28.

Stage 1

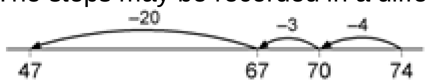
Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

15 - 7 = 8

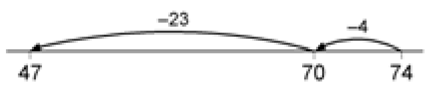


74 - 27 = 47 worked by counting back:

The steps may be recorded in a different order:



or combined:



The notes below give more detail on the counting-up method using an empty number line.

Once the children have a secure understanding of subtraction as taking away in Year 2 then the empty number line as counting on needs to be introduced in small groups. Alongside the introduction of the counting up method the subtraction needs to rewritten as the inverse so children can see how the method relates to the sum.

For example – 45 – 23 = ? which becomes 23 + ? = 45. An example of the method layout can be found in appendix 1.

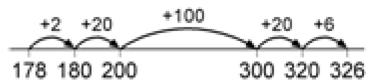
Year 2, 3, 4, 5 and 6

The counting-up method

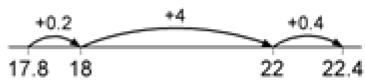
- The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.

Or:

- **With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally.**
- **The most compact form of recording remains reasonably efficient.**



- The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed.
- This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.



Year 5 & 6 – if children are ready, introduce as group work

Stage 3: Expanded layout, leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.

Stage 3

Partitioned numbers are then written under one another:

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- The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.

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Example: 74 - 27

$$\begin{array}{r} 70 + 4 \\ -\ \ 20 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{60}{\cancel{70}} + \overset{14}{\cancel{4}} \\ -\ \ 20 + 7 \\ \hline 40 + 7 \end{array}$$

$$\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$$

Example: 741 - 367

$$\begin{array}{r} 700 + 40 + 1 \\ -\ \ 300 + 60 + 7 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{600}{\cancel{700}} + \overset{130}{\cancel{40}} + \overset{11}{\cancel{1}} \\ -\ \ 300 + 60 + 7 \\ \hline 300 + 70 + 4 \end{array}$$

$$\begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$$

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Written methods for multiplication of whole numbers

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- Add combinations of whole numbers using the column method (see above).

Year 2 – the first method to check for understanding is to use pictures or dots, then the empty number line and by the end of the year when the children are ready introduce Stage 1.

Progression through KS 2

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Stage 1: Mental multiplication using partitioning

- Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.

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Stage 1

Informal recording in Year 4 might be:

$$\begin{array}{ccccl} & & 43 & & \\ 40 & + & & 3 & \\ \downarrow & & & \downarrow & \times 6 \\ 240 & + & & 18 & = 258 \end{array}$$

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Stage 2: The grid method

- As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

Stage 2

38 × 7 = (30 × 7) + (8 × 7) = 210 + 56 = 266

×	7
30	210
8	56
	266

- ***The next step is to move the number being multiplied (38 in the example shown) to an extra row at the top. Presenting the grid this way helps children to set out the addition of the partial products 210 and 56.***
- ***The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.***

	30 + 8
×	7
	210
	56
	266

Stage 5: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

Stage 5

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

	50	6	
×	20	7	
	1000	350	1350
	120	42	162
			1512
			1

- ***Reduce the recording, showing the links to the grid method above.***

56 × 27 is approximately 60 × 30 = 1800.

	56	
×	27	
	1000	50 × 20 = 1000
	120	6 × 20 = 120
	350	50 × 7 = 350
	42	6 × 7 = 42
	1512	
	1	

- Reduce the recording further.
- The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.

56 × 27 is approximately 60 × 30 = 1800.

```
  56
× 27
1120     56×20
 392     56× 7
1512
 1
```

Stage 6: Three-digit by two-digit products

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

Stage 6

286 × 29 is approximately 300 × 30 = 9000.

×	20	9	
200	4000	1800	5800
80	1600	720	2320
6	120	54	174
			8294
			1

Reduce the recording, showing the links to the grid method above.

- This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

```
  286
×  29
 4000    200×20 = 4000
 1600     80×20 = 1600
  120      6×20 =  120
 1800    200× 9 = 1800
  720     80× 9 =  720
   54      6× 9 =   54
 8294
 1
```

- Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
- Again, the carry digits in the partial products are usually carried mentally.

286 × 29 is approximately 300 × 30 = 9000.

```
 286
×  29
5720     286×20
2574     286× 9
8294
1
```

Written methods for division of whole numbers

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division - for example in 18 ÷ 3 = 6,the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally - for example, find the remainder when 48 is divided by 5;
- Understand and use multiplication and division as inverse operations.

Year 2 - Introduce division as grouping using objects and dots. Then progress to using the empty number line and partitioning.

Year 3 – continue with grouping and empty number line. Then once children confident and secure introduce, partitioning and the grid method with links to the grid method of multiplication.

×		
7	70	14

→

×	10	2
7	70	14

10 + 2 = 12

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

Year 3 and Year 4

Stage 1: Mental division using partitioning

- Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient.
- Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.
- Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by6.

Stage 1

One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

Informal recording in Year 4 for 84 ÷ 7 might be:

84
70 + 14
↓ ↓ ÷ 7
10 + 2 = 12

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Another way to record is in a grid, with links to the grid method of multiplication.

×		
7	70	14

→

×	10	2
7	70	14

10 + 2 = 12

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

Also record mental division using partitioning:

$$64 \div 4 = (40 + 24) \div 4$$
$$= (40 \div 4) + (24 \div 4)$$
$$= 10 + 6 = 16$$
$$87 \div 3 = (60 + 27) \div 3$$
$$= (60 \div 3) + (27 \div 3)$$
$$= 20 + 9 = 29$$

Remainders after division can be recorded similarly.

	$$\begin{aligned}96 \div 7 &= (70+26) \div 7\\ &= (70 \div 7) + (26 \div 7)\\ &= 10 + 3\ R\ 5 = 13\ R\ 5\end{aligned}$$
Year 4 and Year 5	
Stage 2: Short division of TU ÷ U • 'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning. • Short division of two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be at the end of Year 4 or the beginning of Year 5. • The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21divided by three?' which gives the answer 7.	***Stage 2*** ***The short division method is recorded like this:*** $\begin{array}{r} 20+7 \\ 3\overline{)60+21} \end{array}$ ***This is then shortened to:*** $\begin{array}{r} 2\ 7 \\ 3\overline{)8\ {}^{2}1} \end{array}$ ***The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript.*** ***The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.***

Year 4 and Year 5

Stage 4: Short division of HTU ÷ U • 'Short' division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. • The accompanying pattern is 'How many threes in 290?' (The answer must be a multiple of 10). This gives 90threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. • Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be at the end of Year	***Stage 4*** For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. $\begin{aligned}291 \div 3 &= (270+21) \div 3\\ &= (270 \div 3) + (21 \div 3)\\ &= 90 + 7\\ &= 97\end{aligned}$ The short division method is recorded like this: $3\overline{)290+1} = \begin{array}{r} 90+7 \\ 3\overline{)270+21} \end{array}$ This is then shortened to: $\begin{array}{r} 9\ 7 \\ 3\overline{)2\ 9\ {}^{2}1} \end{array}$ The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are

5 or the beginning of Year 6.	to be divided by 3. The 97 written above the line represents the answer: 90 + 7,or 9 tens and 7 ones

Year 6

Stage 5: Long division The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.	***Stage 5*** How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 24) 560 20 − 480 24 × 20 80 3 72 24 × 3 8 Answer: 23 R 8 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. 23 24) 560 −480 80 −72 8 Answer: 23 R 8